What is Absolute Phase Angle?

We have received many calls from customers confused about the phase measurements made by the Model 1133A Power Sentinel. Basically, these questions go something like, “Why do the phase measurements made by the 1133A keep changing? I know that my system phase is not changing that much.”

This is because the phase measurements made by the 1133A are absolute phase, that is, phase angles measured with respect to UTC, Coordinated Universal Time as maintained by the BIH in Paris and the US Naval Observatory in Washington, DC, and transmitted to the 1133A via the GPS System. The 1133A measures the phase angle of the three voltage and current signals (and all their harmonics) relative to the 1PPS on-time signal synchronized to within one microsecond of UTC anywhere in the world.

Absolute phase in the 1133A is defined as zero degrees corresponding to the positive maximum of a cosine wave coincident with the on-time reference. If the frequency is above nominal, the phase angle will be increasing over time; if below, it will be decreasing. It will, of course, wrap around at ±180 degrees. Therefore, since there is always some small error in the nominal 50 or 60 Hz input signal, the measured phase angle will always be changing.

Why is Absolute Phase Useful?

Whenever measurements (of any sort) are made with reference to a consistent external standard, whether it is UTC or the international definitions of voltage, resistance, or energy maintained by national laboratories, then it becomes possible to compare measurements made at different times and places and know with certainty how they compare. Relative phase angle measurements (of the type offered by non-synchronized instruments) have very little value in comparing, say, phase angle measurements taken at two different substations. A common frame of reference is lacking. This is what GPS synchronization can provide.

With synchronized measurements, it is now possible to directly compare phase measurements between more than just the voltages and currents measured by a single piece of equipment. For example, it is known that the phase displacement across a transmission line is a measure of the energy being transmitted across the line, or in other words, the stress on the line. Periodic variations in this phase angle, a result of periodically-varying energy flows, are a sensitive indicator of system instabilities [1]. Simply measuring the variations in energy flow provides one data point, and is one indication of instability; knowing the phase variation can help to pinpoint the actual source of the problem.

Converting to Relative Phase

Most ‘traditional’ phase measurements, lacking a reference such as UTC, use the A-phase voltage as their reference. This is by choice, and is arbitrary, but the choice has been a common one – so much so that users have come to expect this as the ‘correct’ phase indication. As a matter of fact, by itself an absolute phase measurement is little more than a laboratory curiosity. We are almost always interested in the phase angle between two things – between the phase voltages of a transmission line; between the voltages and currents; between different places (a measure of power flow, as described above) or different times (a measure of time-related frequency stability). All of this and more can be gotten from absolute phase measurements synchronized to a common time reference.

To compare phase measurements on a system where all the signals are at the same frequency (50 or 60 Hz, in power systems), it is only necessary to subtract the phase measurement of the reference signal from the measurement of interest, then normalizing if required. For example, if you are interested in the relative phase between the A and B voltages, you subtract the A phase measurement from the B phase. If the A phase angle is -160 degrees, and the B phase angle is 79 degrees, then the difference is: 79 - (-160) = 239. If you want an answer normalized between ±180 degrees, adjust by adding or subtracting 360: 239 - 360 = -121.

Harmonics

The other situation that comes up is a relative indication of phase for harmonics. This is a little more complicated, but still not hard to implement. The important realization is that what the 1133A measures is the phase angle, for fundamental and all harmonics, for an arbitrary (relative to the signal phase) time point. What we want to know is the phase angle of the harmonics for a time point corresponding to zero degrees of the signal phase. This is a simple time-shift operation. Phase is proportional to frequency and time: $\theta = wt$. For harmonics, $\theta = nw_o t$, where n is the harmonic number and w_o the fundamental frequency. A little work with this relationship shows that to find the relative phase of a harmonic, it is necessary only to multiply the reference phase angle by the harmonic number before subtraction. It is that simple. Normalization may require multiple additions or subtractions of 360 to get the desired answer for the higher harmonics, but there are a lot of 'clever' (i.e. more computationally-efficient) ways to implement such a calculation in a computer program.

For example, we have found in the implementation of the Model 1133A that the most convenient way to handle phase internally is through complex mathematics. All calculations made in the 1133A's DSP are made with complex (a + bi) quantities; phase angle is not explicitly a part of any calculations except at the very beginning and end of the process. This eliminates all of the phase normalizations which would otherwise be required. At the end of the calculation process, we have a complex number representing (for example) voltage. This can be directly converted into phase angle in radians, using the two-argument arctangent function, and magnitude in volts, using the square root of the sum of the squares of the two parts of the complex number. No normalization is ever required. This is much more efficient computationally and in terms of program length than the otherwise almost endless need for phase normalizations. Additionally, the errors caused by word-length limits (roundoff and truncation) have a less-significant effect on calculations made in this way, resulting in better accuracy and lower noise.

A phase angle correction in this nomenclature is represented by a vector (complex number) having a magnitude of 1 and the desired phase angle; in other words, it is $\cos(\theta) + i * \sin(\theta)$. This number is raised to the desired power (equal to the harmonic number) to obtain the correction for each harmonic. Since the harmonics are typically done in sequence, this is most easily done with a regressive implementation: each iteration, for the next harmonic value, the previous (complex) correction result Θ^n is (complex) multiplied by θ (complex) to obtain $\Theta^{(n+1)}$. This result is multiplied by the (complex) harmonic magnitude to correct for the phase variation. You can see with a little further thought that this same operation can also convert the harmonic magnitude from volts or amperes to a relative indication, such as percent or P/U, simply by starting the regressive operation with the correct value, 100/(rms fundamental) for example. This can be done with an absolute minimum of additional calculation, in particular avoiding multiple computation-intensive divisions, and is an example of the power of complex mathematics in such applications.

Phase Variations with Time

Again from the relationship $\theta = wt$, we can see that a change in phase from one point in time to another is a result of a frequency difference. More exactly, the phase angle is continually rotating in time, completing 50 or 60 complete cycles per second in a power system. However, by measuring phase angle at a sampling rate which is many times below the Nyquist rate, and specifically an integer number of times less than the nominal frequency, we can in effect 'alias' the phase angle measurements. The result of this, for a sample rate of (for example) 1 per second, or 10 per second, is that the phase angle does not change unless there is a frequency error. We are sampling the phase function at (nominally) the same point every time; any phase variation is due to a frequency error.

We must of course be very careful with any sort of sub-sampling technique, because much information about the signal is being discarded. We are in essence making the assumption that the signal frequency is close to nominal. If it is not, we may get an answer which is grossly incorrect, and the longer the time interval between the phase angle measurements, the smaller the frequency measurement range and the greater the likelihood of error. However, in modern power systems, the frequency does not vary all that much, and if proper care is taken this calculation may be performed externally to the 1133A.

In the Model 1133A, all internal measurements of frequency and time deviation are made using phase measurements. These calculations are performed internally at a rate of 20/second, allowing a frequency offset of ±10 Hz before problems set in. This problem manifests itself in the inability to tell the difference between a frequency error of (e.g.) +11 Hz and –9 Hz, at least without reference to additional information. This is adequate for all conceivable conditions in a power system.

It is also possible to perform these calculations externally using the phase numbers, provided that the sample rate vs. frequency error concerns described above are taken into account. This is normally most useful for reconstructing data after the fact, when a reduced data set was stored in order to save storage space or data transmission bandwidth. Otherwise, one might as well let the 1133A do the work. Within stated limitations, the concept is sound in any event.

Another interesting use of phase information is to look at the time variation of relative phase numbers. As mentioned earlier and described in [1], this is a useful way to investigate power system loading and stability.

Phasors

Phasors are defined in IEEE Standard 1344-1995 [2], where they are more precisely known as 'synchrophasors.' They are also discussed in [1] and several other articles in the trade press. Phasor representation of a three-phase bus consists of six complex numbers, one each for the three bus voltages and the three line currents. In IEEE 1344, the complex numbers are formatted for data transmission as 16-bit integers for each of the components, somewhat reducing resolution from that which is maintained internally in the 1133A, which uses 32-bit floating point values for everything. However, this resolution is adequate for the intended purposes, which is real-time gathering and analysis of data from numerous, widely-spaced locations, which requires minimum data bandwidth. The goal of phasor analysis is prediction and control of transient disturbances and oscillatory behaviors on the power grid, with the intended result being to improve overall system capacity and reliability. Phasors can also be used for system protection and relaying, although such applications are presently in the research phase.

For phasors to provide useful information, the instrument making the measurements must have an absolute time reference, so that measurements made at different locations can be compared. A good portion of IEEE 1344-1995 is devoted to time synchronization requirements to meet this need. The Arbiter Systems GPS Satellite-Controlled Clocks were the first timing products in the industry to fully meet the requirements of IEEE 1344-1995.

The Model 1133A Power Sentinel, with built-in GPS timing, is ideal for the measurement of phasors. Phasor output is a simple formatting operation for the 1133A, since the internal representation of voltage and current is already in GPS-synchronized, complex-vector form. The 1133A measures phasors and formats the results in accordance with IEEE-1344 with no additional hardware required, and therefore it is a very cost-effective solution. The phasor update rate of the 1133A is 20 per second.

Summary

Absolute phase angle measurements provide the power engineer with a powerful new tool to compare measurements made at multiple locations and times. Before the advent of GPS synchronized, low-cost, multifunction instruments, this was possible only with great difficulty and expense. Using simple calculations, any two-phase measurements made with the Arbiter Systems Model 1133A Power Sentinel may now be compared, and the relative phase easily determined. The results are available in a variety of formats, including the IEEE Standard 1344-1995 Synchrophasor format.

[1] See, for example, Richard P. Schulz and Beverly B. Laios, "Triggering Tradeoffs for Recording Dynamics," *IEEE Computer Applications in Power*, April 1997, pp. 44 ff. This paper also includes several further references to the prior art.

[2] IEEE Standard 1344-1995, IEEE Standard for Synchrophasors for Power Systems, The Institute of Electrical and Electronic Engineers, Inc., Piscataway, New Jersey, 1995.